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Generating Portable High-Performance Code via Multi-Dimensional Homomorphisms

An Algebraic Approach toward Performance, Portability, and Productivity for Data-Parallel Computations on Multi- and Many-Core Architectures

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Motivation

Observation:

Applications

Linear Algebra (BLAS)

GEMM GEMV

DOT

… 152 routines!

Tensor Contractions

…

Stencils

…

X

Architectures

X

Input Sizes

Machine Learning

…

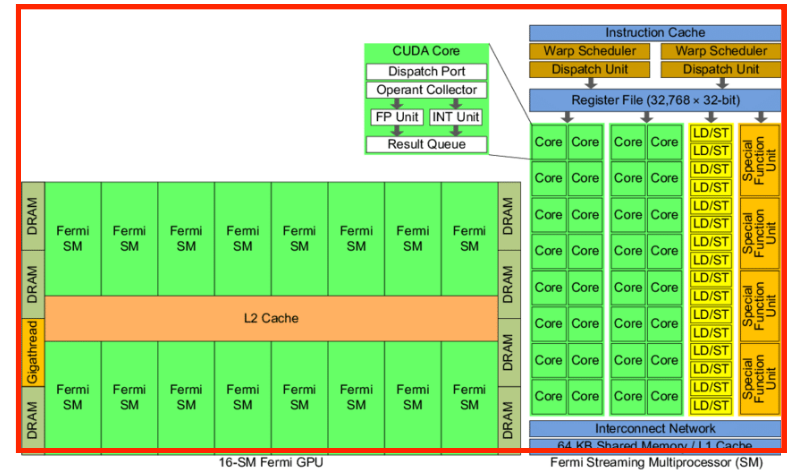
Numerical Computations

…

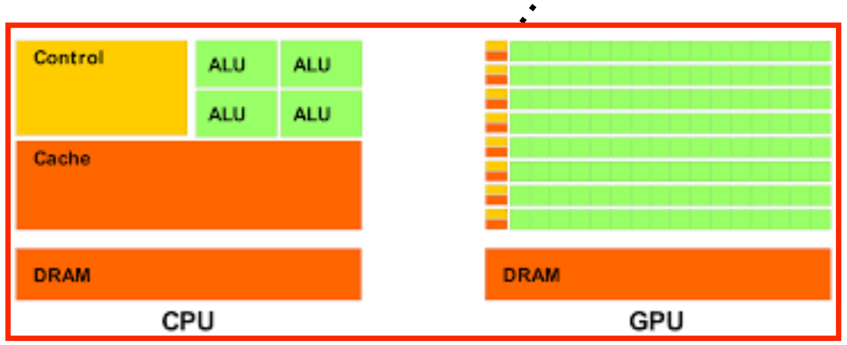
Combinatorial Explosion

Motivation

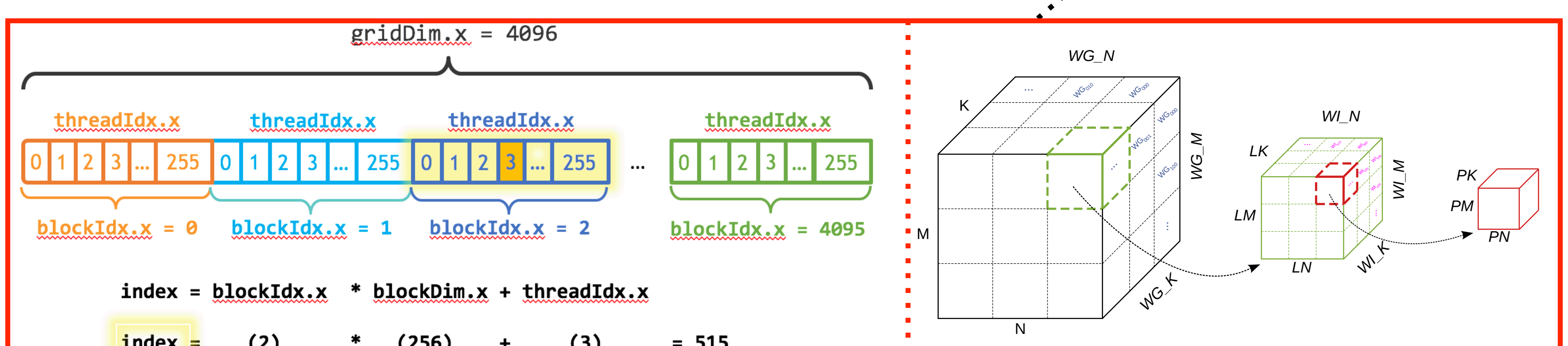
<u>In a perfect world:</u>



Write **one piece of code** for our application that provides **high performance**, is **performance-portable over different architectures** and **input sizes** and that is **easy to implement**.



dimensions			$C = AB$	$C = A^T B^T$	small = 1
m	n	k	Shape	Shape	
large	large	large			gemm
large	large	small			ger
large	small	large			gemv
large	small	small			axpy $(\beta = 1)$
small	large	large			gemv
small	large	small			axpy $(\beta = 1)$
small	small	large			dot $(\alpha = \beta = 1)$
small	small	small			scalar mult.

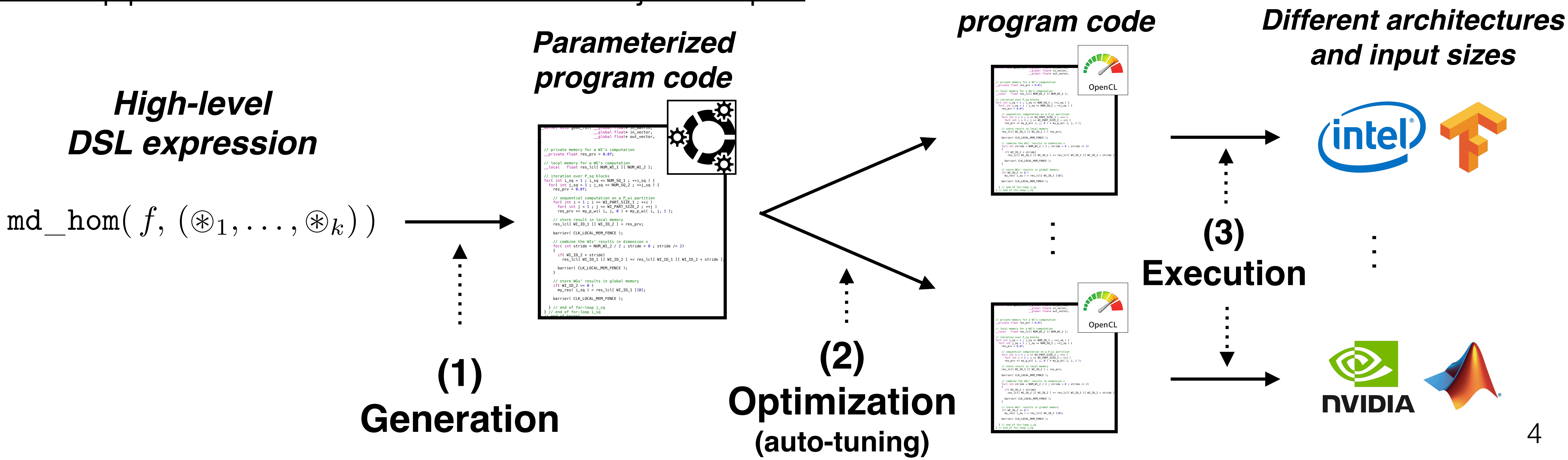


Our Approach

<u>We provide an approach to address all these challenges for our class of *Multi-Dimensional Homomorphisms*:</u>

- **Multi-Dimensional Homomorphisms (MDHs)** are **formally-defined** class of functions that **cover broad range of data-parallel computations**, e.g.: linear algebra (BLAS), stencils computations, …
- We **enable conveniently implementing MDHs** by providing a **high-level DSL** for them.
- We provide a **DSL compiler** that **automatically generates OpenCL code** so that we can target different parallel architectures (e.g., CPU and GPU).
- Our OpenCL code is **fully automatically optimizable** (auto-tunable) — for each combination of a **target architecture**, and **input size** — by being generated as targeted to **OpenCL's abstract device models** and as **parametrized in these models' performance-critical parameters.**

<u>Our approach consists of three major steps:</u>



Agenda

We discuss the three major steps of our approach:

1. **Generation**

2. **Optimization**

3. **Execution**

Afterwards:

4. **Experimental Results**

5. **Current/Future Work**

Multi-Dimensional Homomorphisms

Our class of targeted computations is formally defined as:

Definition: [*Multi-Dimensional Homomorphisms [1]*]

Let T and T' be two arbitrary types. A function $h : T[\,N_1\,]\ldots[\,N_d\,] \to T'$ on d-dimensional arrays is called a *Multi-Dimensional Homomorphism (MDH)* iff there exist *combine operators* $\circledast_1, \ldots, \circledast_d : T' \times T' \to T'$, such that for each $k \in [1, d]$ and arbitrary, concatenated input MDA $a +\!\!+_k\ b$:

$$h(\ a +\!\!+_k\ b\ )\ =\ h(a)\ \circledast_k\ h(b)$$

Definition: [`md_hom`]

We write

$$\texttt{md\_hom}(\ f, (\circledast_1, \ldots, \circledast_d)\ )$$

for the unique d-dimensional homomorphism with combine operators $\circledast_1, \ldots, \circledast_d$ and action f on singleton arrays.

[1] Rasch et al. "Multi-Dimensional Homomorphisms and Their Implementation in OpenCL." *International Journal of Parallel Programming, 2018*

MDH — Examples

Important functions are MDHs — we can express them conveniently in our DSL:

Linear Algebra

```
GEMM = md_hom( *, (++, ++, +) ) o view( A,B )( i,j,k )( A[i,k], B[k,j] )
```

Matrix Multiplication

What's happening?

1. Prepare the domain-specific input for `md_hom`; for this, our DSL provides pattern `view`.
   - here: fuse matrices A and B to 3-dimensional array of pairs consisting of the elements in A and B to multiply: `i,j,k ↦ (A[i,k],B[k,j])`.
2. Apply multiplication (denoted as `*`) to each pair.
3. Combine results in dimension `k` by addition (`+`).
4. Combine results in dimensions `i` and `j` by concatenation (`++`).

MDH — Examples

Important functions are MDHs — we can express them conveniently in our DSL:

Linear Algebra

```
GEMM = md_hom( *, (++, ++, +) ) o view( A,B )( i,j,k )( A[i,k], B[k,j] )
GEMV = md_hom( *, (++,     +) ) o view( A,B )( i,  k )( A[i,k], B[k]   )
DOT  = md_hom( *, (        +) ) o view( A,B )(     k )( A[k]  , B[k]   )
```

- Matrix-Vector Multiplication (`GEMV`) and Dot Product (`DOT`) are expressed similarly to `GEMM`.
- Both are special cases of `GEMM`:
  - GEMV: `B` is `Kx1` matrix (i.e., no dimension `j`);
  - DOT: `A` is `1xK` matrix, B is `Kx1` matrix (i.e., no dimensions `i` and `j`).
- Note:

  `GEMV` and `DOT` are not required to be expressed in our approach → `GEMM` *provides same high performance, because it can be optimized for very irregular input sizes!*

MDH — Examples

Important functions are MDHs — we can express them conveniently in our DSL:

Linear Algebra

```
GEMM = md_hom( *, (++, ++, +) ) o view( A,B )( i,j,k )( A[i,k], B[k,j] )
GEMV = md_hom( *, (++,     +) ) o view( A,B )( i,  k )( A[i,k], B[k]   )
DOT  = md_hom( *, (        +) ) o view( A,B )(     k )( A[k]  , B[k]   )
```

```
Access neighboring elements
 within their input buffer
```

Stencil Computations

```
Gaussian_2D = md_hom( G_func, (++,++)     ) o view(…)
Jacobi_3D   = md_hom( J_func, (++,++, ++) ) o view(…)
```

Data Mining

PRL = md_hom(weight, (++, $\otimes_{max}$)) o view(…)

```
Has user-defined combine operator that
  operates on user-defined data type
```

```
Often very high dimensional
      (e.g., 7 dims)
```

Machine Learning

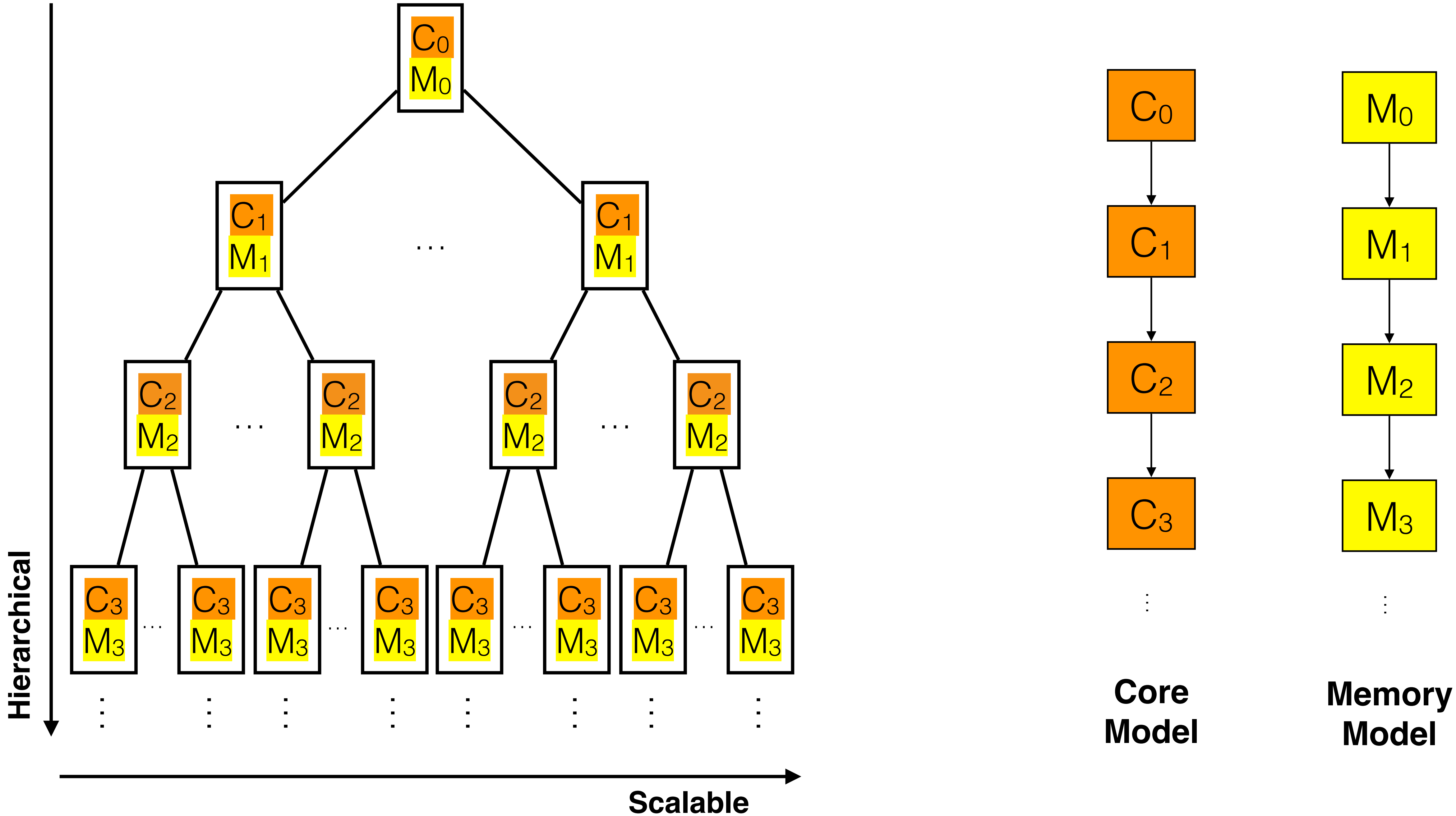
```
TC = md_hom( *, (++,…,++ , +,…,+) ) o view(…)
```

Further examples: MLP, SVM, ECC, …, Mandelbrot, Parallel Reduction, …

Our DSL needs only two patterns:
`md_hom(…)` and `view(…)`

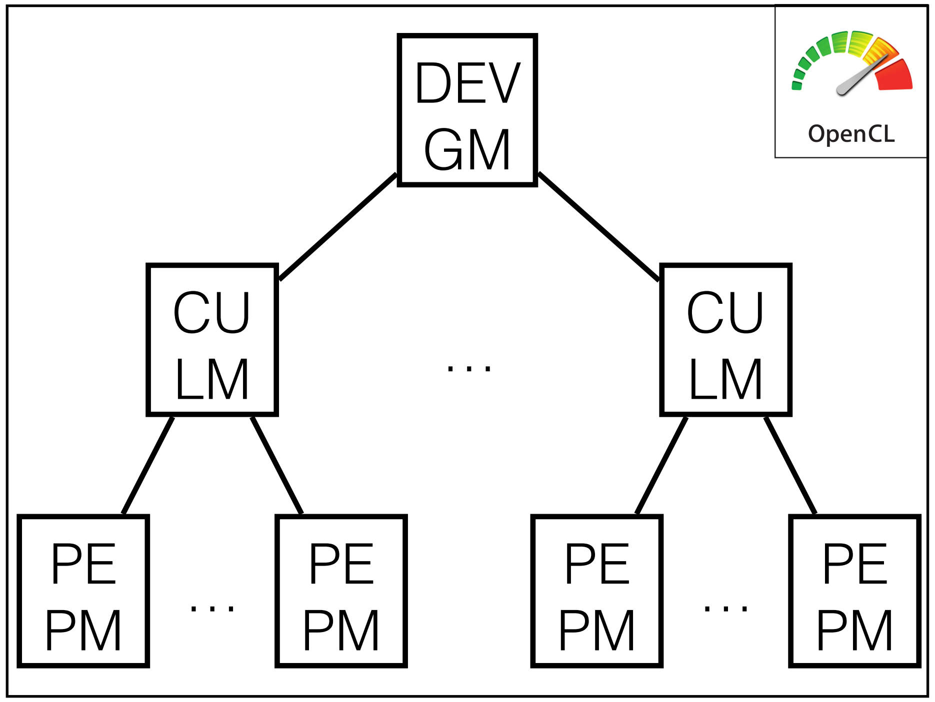
MDH — Target Machine Model

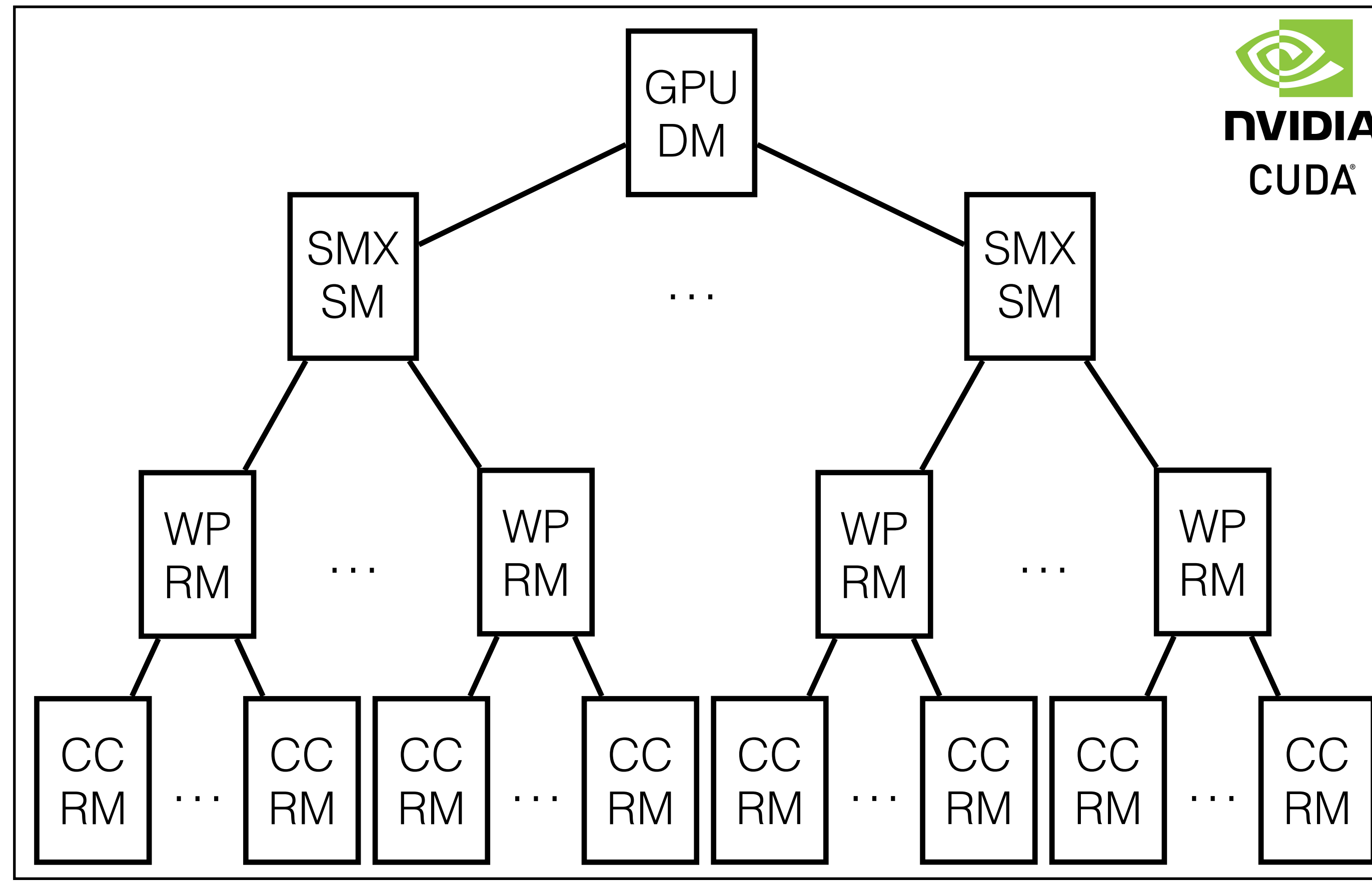
<u>We target with MDHs ***hierarchical***, ***scalable*** *machine models*:</u>

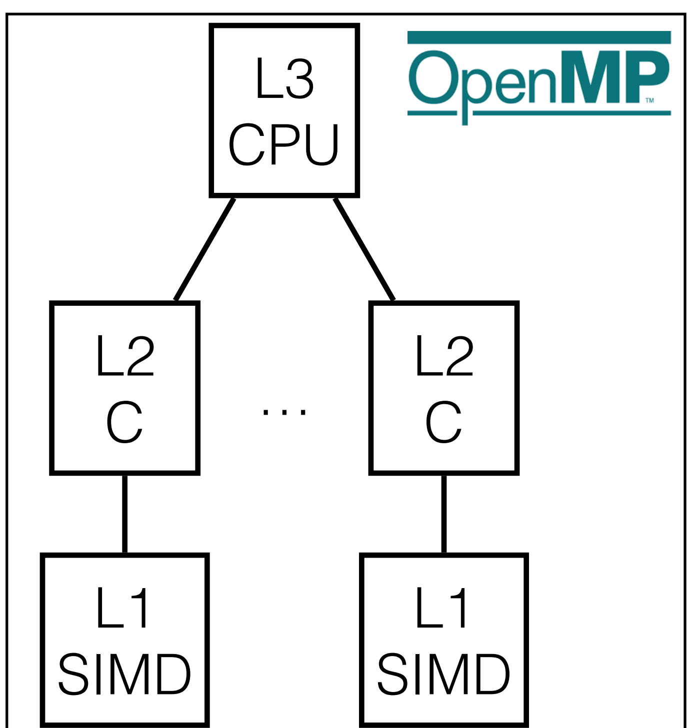


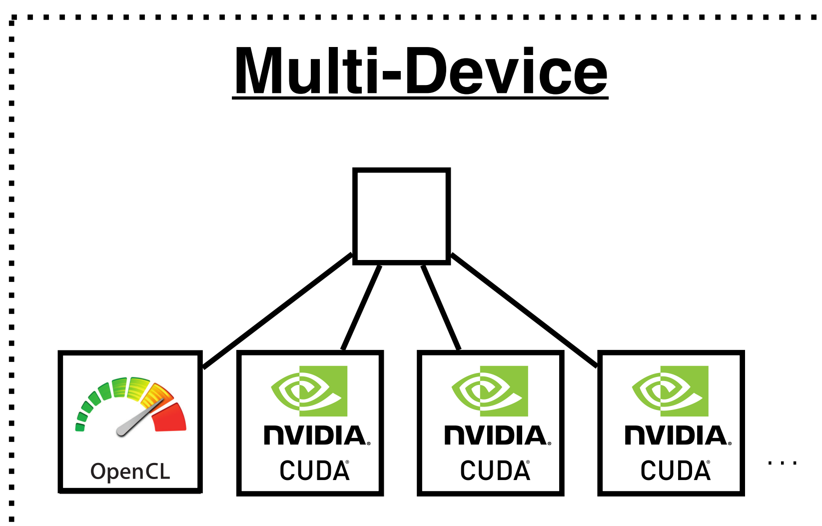
MDH — Target Machine Model

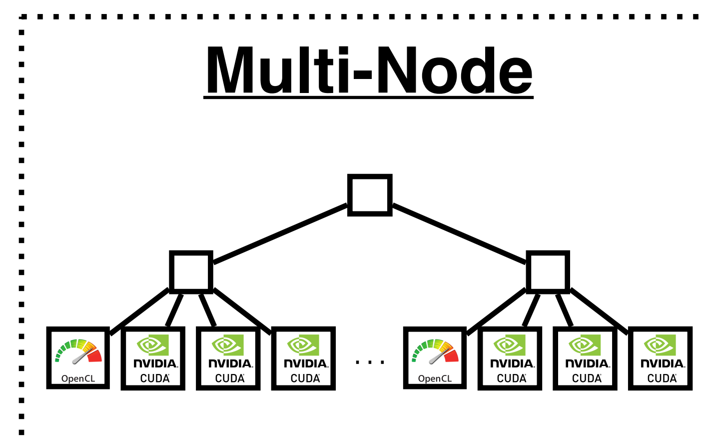
Examples of such *machine models*:











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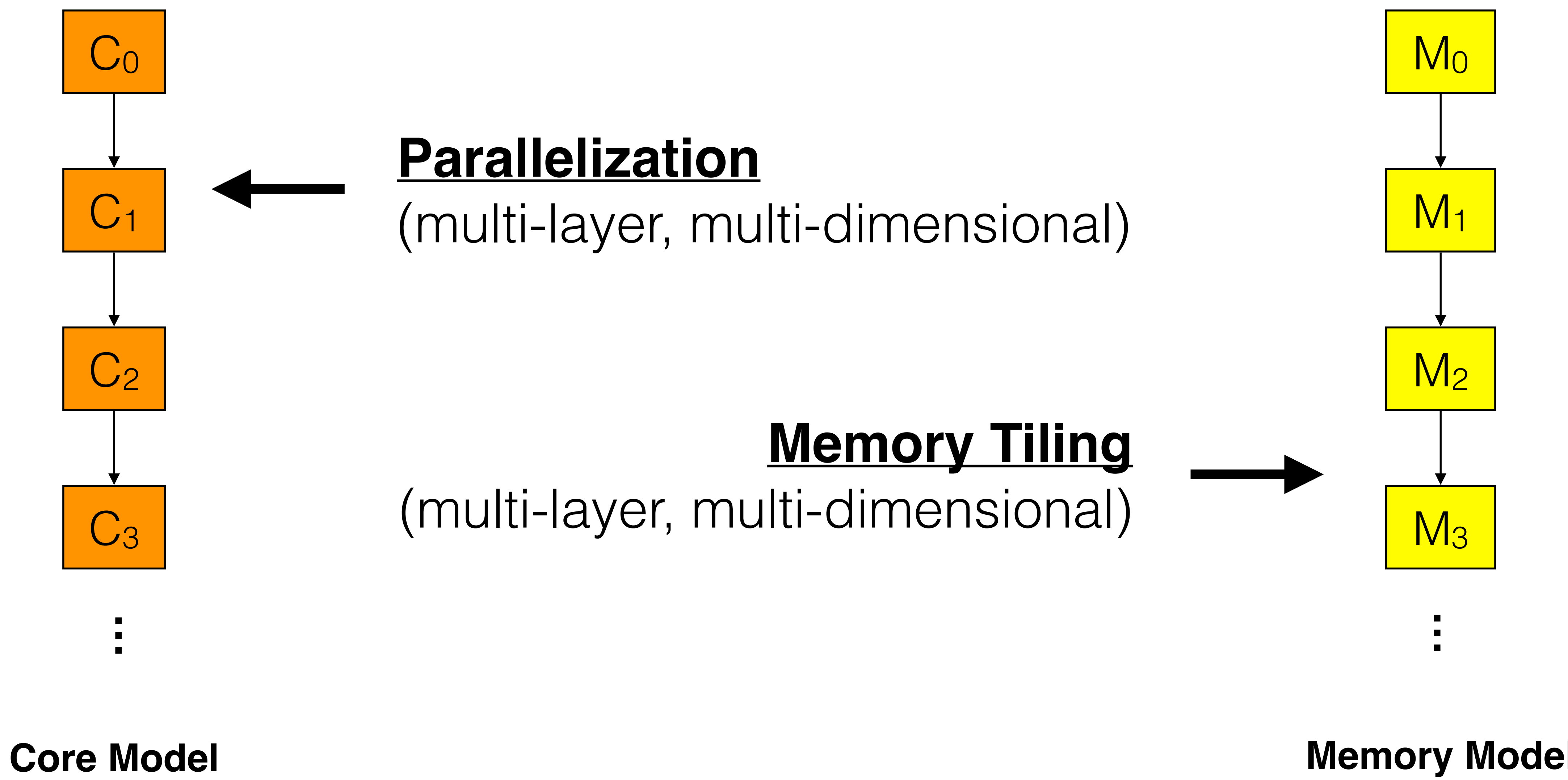
Code Generation for MDHs

Our **uniform `md_hom` representation** of MDHs enables **systematically generating code for such machine models,** which can be **automatically optimized (auto-tuned)**:



<u>In the following:</u> Explain our code generation at example of OpenCL.

Code Generation for MDHs

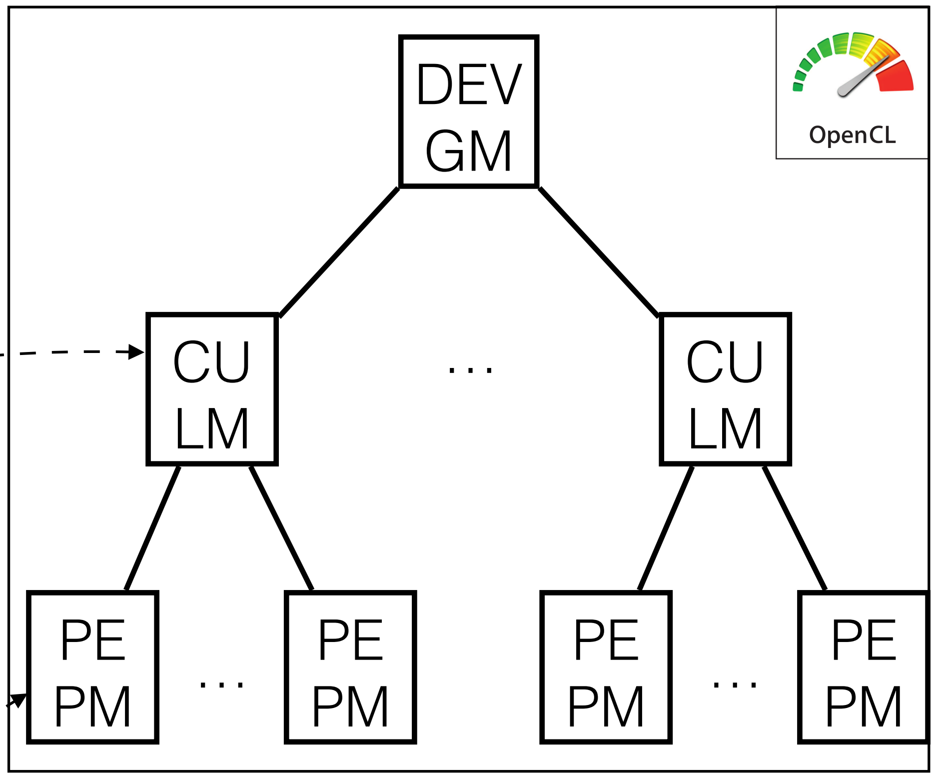
1. Parallelization (multi-layer, multi-dimensional):

```
#reduce ⊛_1
for( i_1 = 1, ... , N_1 )
  ⋱
  #reduce ⊛_d
  for( i_d = 1, ... , N_d )
  {
     f( a[i_1]...[i_d] )
  }
```

MDH pseudocode for $\texttt{md\_hom}(\, f,\ (\circledast_1, \ldots, \circledast_d)\ )$

```
#reduce ⊛_1
parallel_for( i_1 = 1, ... , NUM_WG_1 )
  ⋱
  #reduce ⊛_d
  parallel_for( i_d = 1, ... , NUM_WG_d )
```

```
#reduce ⊛_1
parallel_for( ii_1 = 1, ... , NUM_WI_1 )
  ⋱
  #reduce ⊛_d
  parallel_for( ii_d = 1, ... , NUM_WI_d )
```



- We parallelize for each of OpenCL's **two parallel layers.**
- We parallelize **on each layer** in **all *d* dimensions** of the MDH.
- <u>We **auto-tune** the number of threads **on each layer** and **in each dimension**.</u>

Combine operators not necessarily concatenation

Code Generation for MDHs

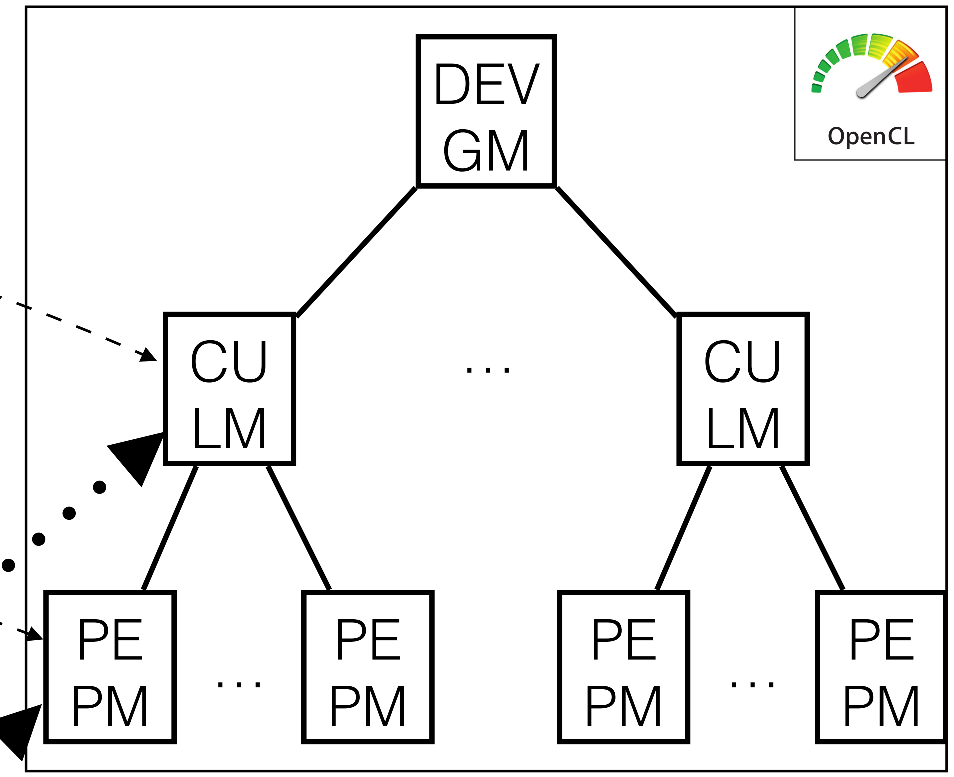
2. Memory Tiling (multiple layers, multiple dimensions):

```
#reduce ⊛1
parallel_for( i_1 = 1, ... , NUM_WG_1 )
  ⋱
  #reduce ⊛d
  parallel_for( i_d = 1, ... , NUM_WG_d )
```

Work-Groups

```
#reduce ⊛1
parallel_for( ii_1 = 1, ... , NUM_WI_1 )
  ⋱
  #reduce ⊛d
  parallel_for( ii_d = 1, ... , NUM_WI_d)
```

Work-Items



LM-Tiles

PM-Tiles

```
for( j_1 = 1, ... , NUM_LM_TL_1 )
  ⋱
  for( j_d = 1, ... , NUM_LM_TL_d )
```

```
for( j_1 = 1, ... , NUM_PM_TL_1 )
  ⋱
  for( j_d = 1, ... , NUM_PM_TL_d )
{
  f( ...)
}
```

- We tile for each of OpenCL's **two memory layers.**
- We tile **on each layer** in **all dimensions** of the MDH.

‣ We **auto-tune** the sizes of tiles **on each layer** and **in each dimension**.

Code Generation for MDHs

Our OpenCL implementation is generated as **parametrized in performance-critical parameters** (a.k.a. tuning parameters) of OpenCL's abstract device models:

No.	Name	Description
1	$\texttt{NUM\_WG}^{\texttt{<i>}}$	number of Work-Groups
2	$\texttt{NUM\_WI}^{\texttt{<i>}}$	number of Work-Items
3	$\texttt{LT\_SIZE}^{\texttt{<i>}}$	local tile size
4	$\texttt{PT\_SIZE}^{\texttt{<i>}}$	private tile size
5	$\texttt{MEM\_INP}^{\texttt{<LYR,b,i>}}$	memory regions for caching input
6	$\texttt{MEM\_RES}^{\texttt{<LYR,b,i>}}$	memory regions for comp. results
7	$\sigma^{\texttt{<LYR>}}_{\texttt{arr}\to\texttt{ocl}}$	mapping array to OpenCL dimensions
8	$\sigma^{\texttt{<LYR,b>}}_{\texttt{buff-do}}$	buffer dimension order
9	$\sigma^{\texttt{<LYR>}}_{\texttt{mdh-do}}$	MDH dimension order
10	CMB_RES	layer to combine results on

We have chosen all parameters independently of a target: MDH, architecture, input size !

Automatic Performance Tuning

We use our ***Auto-Tuning Framework (ATF)*** **[1]** to automatically choose optimized values of our performance-critical parameters.

	Domain-specific auto-tuning	OpenTuner	CLTune	ATF
Arbitrary Programming Language		✓		✓
Arbitrary Application Domain		✓	✓	✓
Arbitrary Tuning Objective	✓	✓		✓
Arbitrary Search Technique	✓	✓	✓	✓
Interdependent Parameters	✓		✓	✓
Large Parameter Ranges	✓	✓		✓
Directive-Based Auto-Tuning				✓
Automatic Cost Function Generation	✓		✓	✓

ATF combines major advantages over state-of-the-art auto-tuning approaches

[1] Rasch et al. "ATF: A Generic, Directive-Based Auto-Tuning Framework." *Concurrency and Computation: Practice and Experience, 2019*

Automatic Performance Tuning

ATF usage: We annotate our generated program code with easy-to-use *tuning directives.*

```
#atf::tp name       NUM_WG_1
         range      interval<int>( 1, N_1 )

#atf::tp name       NUM_WI_1
         range      interval<int>( 1, N_1 )

// …

#atf::tp name       LM_SIZE_1
         range      interval<int>( 1, N_1 )
         constraint LM_SIZE_1 <= N_1

#atf::tp name       PM_SIZE_1
         range      interval<int>( 1, N_1 )
         constraint PM_SIZE_1 <= LM_SIZE_1

// …

// OpenCL kernel code
```

→ **ATF automatically determines optimized parameter values**

(ATF is also available as C++/Python programming library for online auto-tuning)

Execution

We execute our *generated* and *optimized* OpenCL code using our own ***dOCAL*** **[1]** framework which:

1. provides **high-level abstractions** for simplifying implementing **OpenCL host code**, especially for multi-device systems (e.g., by automatically performing memory allocations and synchronization);

2. provides **asynchronous computation efficiency** (e.g., overlapping data transfers and/or kernel computations) by generating and maintaining a data-dependency graph transparently from the user;

3. enables conveniently executing OpenCL kernels on **remote nodes** (via *Boost.Asio*).

[1] Rasch et al., “dOCAL: High-Level Distributed Programming with OpenCL and CUDA”, *The Journal of Supercomputing, 2019*

Execution

Illustration: using dOCAL for executing OpenCL code on GPU.

```
#include "docal.hpp"

int main()
{
  // 1. choose device
  auto device = docal::get_device( "NVIDIA", "Tesla" );

  // 2. declare kernel
  docal::kernel GEMM = docal::source( /* OpenCL Code */ );

  // 3. prepare kernels' inputs
  docal::buffer<float> A( N*N );
  docal::buffer<float> B( N*N );
  docal::buffer<float> C( N*N );

  std::generate( A.begin(), A.end(), std::rand );
  std::generate( B.begin(), B.end(), std::rand );

  // 4. start device computations
  device( GEMM                                          )
        ( nd_range( /* GS */), nd_range( /* LS */ ) )
        ( read ( A ), read( B ), write( C )          );

  // 5. print result
  for( int i = 0 ; i < N*N ; ++i )
    std::cout << C[ i*N + j ];
}
```

OpenCL

Experimental Results

We evaluate or **automatically-generated and auto-tuned code** using:

Applications

1. Linear Algebra Routines: GEMM, GEMV
2. Stencils: Gaussian Convolution 2D, Jacobi 3D
3. Data Mining: Probabilistic Record Linkage
4. Machine Learning: Tensor Contractions

Competitors

- Performance-Portable approaches (e.g., Lift)
- Domain-specific, hand-optimized approaches (Intel MKL, NVIDIA cuBLAS)

Architectures

- Intel Xeon multi-core CPU (E5-2640)
- NVIDIA Tesla V100 GPU (SMX2-16GB)

Data Sets

- RW: Real-world sizes, e.g., from deep learning
- PC: Sizes that are preferable for our competitors

Experimental Results

Linear Algebra

CPU	GEMM		GEMV	
	RW	PC	RW	PC
Lift [1]	fails	3.04	1.51	1.99
MKL	4.22	0.74	1.05	0.87

GPU	GEMM		GEMV	
	RW	PC	RW	PC
Lift [1]	4.33	1.17	3.52	2.98
cuBLAS	2.91	0.83	1.03	1.00

[1] Steuwer et. al, "Lift: A Functional Data-Parallel IR for High-Performance GPU Code Generation", **CGO'17**.

Data Mining

CPU	Probabilistic Record Linkage					
	2^{15}	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}
EKR [5]	1.87	2.06	4.98	13.86	28.34	39.36

[5] Forchhammer et al. “Duplicate Detection on GPUs.” HPI Future SOC Lab, 2013.

Our MDH approach achieves in most cases better performance than our competitors

Stencils

CPU	Gaussian (2D)		Jacobi (3D)	
	RW	PC	RW	PC
Lift [2]	4.90	5.96	1.94	2.49
MKL-DNN	6.99	14.31	N/A	N/A

GPU	Gaussian (2D)		Jacobi (3D)	
	RW	PC	RW	PC
Lift [2]	2.33	1.09	1.14	1.02
cuDNN	3.78	19.11	N/A	N/A

[2] Hagedorn et. al, "High Performance Stencil Code Generation with LIFT.", **CGO'18 (Best Paper Award)**.

Tensor Contractions

GPU	Tensor Contractions								
	RW 1	RW 2	RW 3	RW 4	RW 5	RW 6	RW 7	RW 8	RW 9
COGENT [3]	1.26	1.16	2.12	1.24	1.18	1.36	1.48	1.44	1.85
F-TC [4]	1.19	2.00	1.43	2.89	1.35	1.54	1.25	2.02	1.49

[3] Kim et. al. "A Code Generator for High-Performance Tensor Contractions on GPUs.", **CGO'19**.

[4] Vasilache et al. "Tensor Comprehensions: Framework-Agnostic High-Performance Machine Learning Abstractions." *arXiv,* 2018.

Experimental Results

Summary:

- **Lift:**

  Speedups of up to

  - **1.02x - 5.96x** on own **stencil** samples presented at **[CGO'18 — Best Paper]**;
  - **1.17x - 4.33x** on own **BLAS** samples presented at **[CGO'17]**.

- **COGENT / F-TC:**

  Speedups of **1.19x - 2.89x** on own **tensor contractions** samples **[CGO'19, arXiv'18]**.

- **Intel MKL / NVIDIA cuBLAS:**

  Speedups for **BLAS** of **0.74x-1.00x** on **their preferable sizes**;
  1.03x-4.22x on **real-world input data**.

Experimental Results

Our better results are because:

We generate our OpenCL implementation as **auto tunable** in performance-critical parameters of **OpenCL's abstract device models**.

Lift

Relies on transformation rules which require **hand pruning** optimization space for exploration

EKR Java

Auto-tunable only for architecture.

Intel MKL/MKL-DNN & NVIDIA cuBLAS/cuDNN

Use **hand-crafted heuristics**, rather than auto-tuning.

Tensor Comprehensions & COGENT

Use **less tuning parameters** and **smaller parameter ranges**.

Conclusion

We provide **performance**, **portability**, and **productivity** for MDH functions:

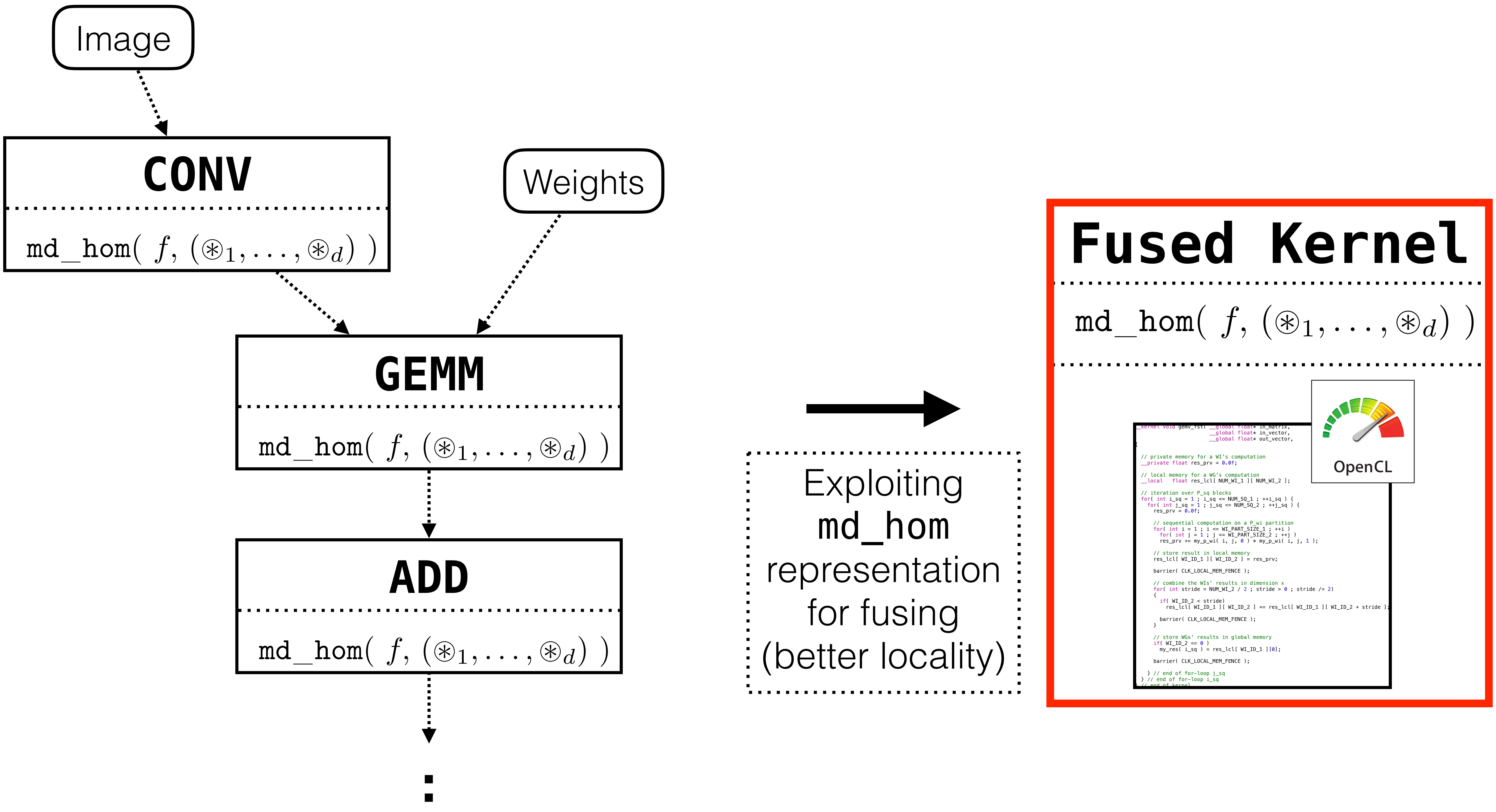
1. MDHs cover a **broad range of applications** (BLAS, Stencil, PRL, TC, …).
2. Our implementation of MDHs provides **high performance** (speedups up to >5) on both CPU and GPU.
3. MDHs are functionally and performance **portable** over architectures and input sizes.
4. MDHs can be **conveniently implemented** using our DSL for MDHs.

Moreover:

- Our **Auto-Tuning Framework (ATF)** is a **general-purpose approach** that **supports auto-tuning of interdependent tuning parameters.**
- We provide our **dOCAL** framework for **conveniently executing OpenCL kernels on multi-device/multi-node systems**, and it automatically **provides asynchronous computation efficiency**.

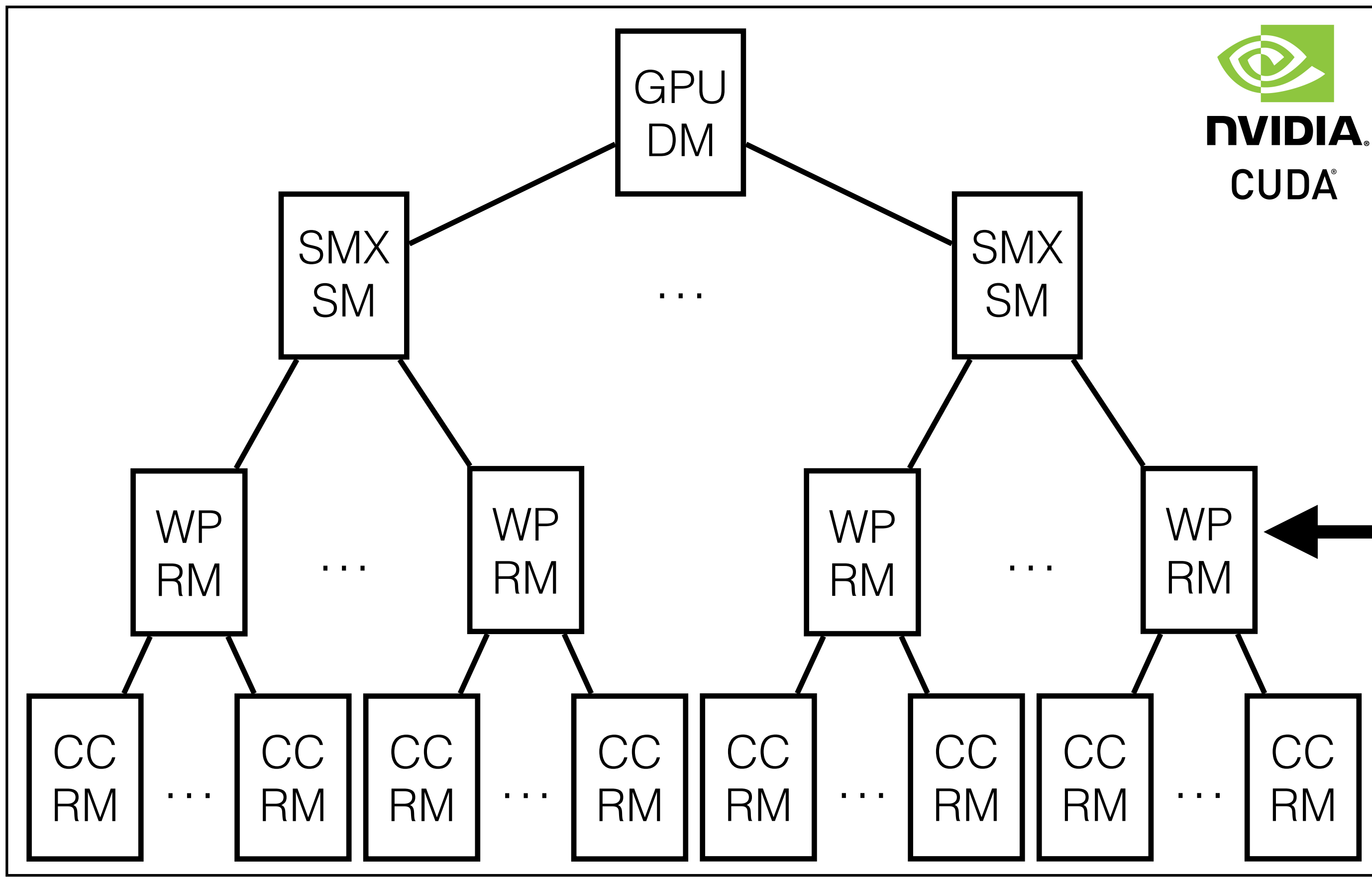
Current/Future Work

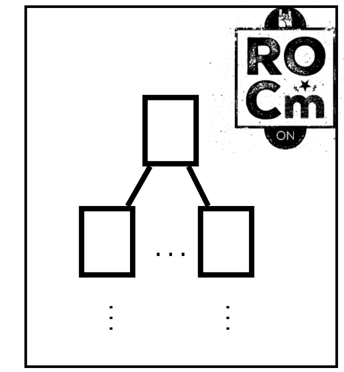
Code generation for composed `md_hom` expression (e.g. as in deep learning graphs):



Current/Future Work

Further backends:

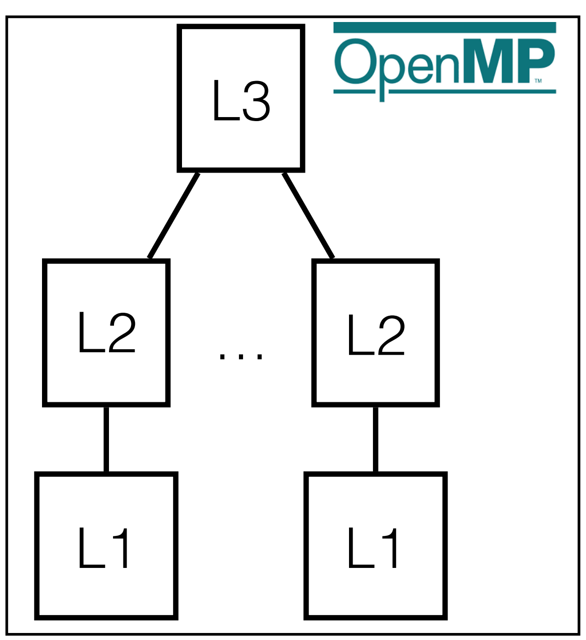




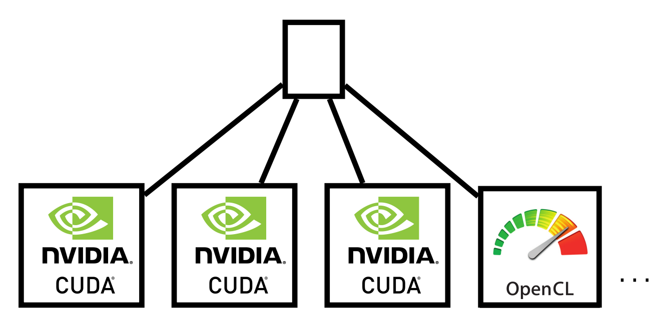
- **Shuffle Operations**
- **Tensor Cores**

Assembler Backends

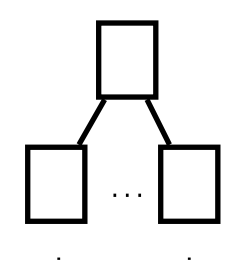
(e.g., NVIDIA PTX, Intel x86-64, …)



Multi-Device



Multi-Node



Current/Future Work

Parallelizing sequential C programs via *Directives*:

```
int main()
{
  // ...

  #pragma mdh ( ,, +:C[i][j] )
  for( int i = 0 ; i < M ; ++i )
    for( int j = 0 ; j < N ; ++j )
      for( int k = 0 ; k < K ; ++k )
      {
        C[ i ][ j ] += A[ i ][ k ] * B[ k ][ j ];
      }

  // ...
}
```

MDH Compiler →

OpenCL

OpenMP

ROCm

NVIDIA CUDA

...

md_hom expression and **view** are automatically derived by MDH compiler:

```
md_hom( loop_body, (++, ++, +:C[i][j]) )
view( A,B )( i,j,k )=( A[i][k], B[k][j])
```

Annotating sequential C-Code with simple *MDH directives* *(similarly as in OpenMP/OpenACC).*

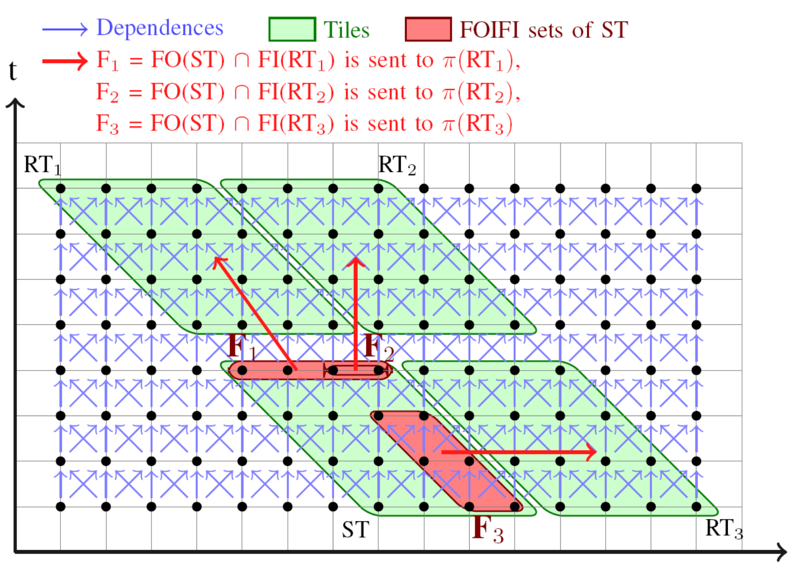
Current/Future Work

Automatic parallelization of sequential C programs:

```
int main()
{
  // ...

   for( int i = 0 ; i < M ; ++i )
    for( int j = 0 ; j < N ; ++j )
      for( int k = 0 ; k < K ; ++k )
      {
        C[ i ][ j ] += A[ i ][ k ] * B[ k ][ j ];
      }

  // ...
}
```



Polyhedral Model

MDH
Code Generation

. . .

Exploiting polyhedral model for MDH code generation

Current/Future Work

Analyze our MDH approach's efficiency for more applications:

Benchmark	Typical Bottleneck of an Unoptimized Implementation	Optimizations Applied	Optimized Implementation Bottleneck	Potential Improvements
cutcp	Contention, Locality	Scatter-to-Gather, Binning, Regularization, Coarsening	Instruction Throughput	Minimizing Reads/Checks of Irrelevant Bin Data
mri-q	Poor Locality	Data Layout Transformation, Tiling, Coarsening	Instruction Throughput	
gridding	Contention, Load Imbalance	Scatter-to-Gather, Binning, Compaction, Regularization, Coarsening	Instruction Throughput	Minimizing Reads/Checks of Irrelevant Bin Data
sad	Locality	Tiling, Coarsening	Memory Bandwidth/Latency	Target Devices with Higher Register Capacities
stencil	Locality	Coarsening, Tiling	Bandwidth	
tpacf	Locality, Contention	Tiling, Privatization, Coarsening	Instruction Throughput	
lbm	Bandwidth	Data Layout Transformation	Bandwidth	
sgemm	Bandwidth	Coarsening, Tiling	Instruction Throughput	
spmv	Bandwidth	Data Layout Transformation	Bandwidth	
bfs	Contention, Load Imbalance	Privatization, Compaction, Regularization	Bandwidth	Avoiding Global Barriers / Better Kernels for Midsized Frontiers
histogram	Contention, Bandwidth	Privatization, Scatter-to-Gather	Bandwidth	Reducing Reads of Irrelevant Input (alleviated by cache)

Parboil

TABLE I
RODINIA APPLICATIONS AND KERNELS (*DENOTES KERNEL).

Application / Kernel	Dwarf	Domain
K-means	Dense Linear Algebra	Data Mining
Needleman-Wunsch	Dynamic Programming	Bioinformatics
HotSpot*	Structured Grid	Physics Simulation
Back Propagation*	Unstructured Grid	Pattern Recognition
SRAD	Structured Grid	Image Processing
Leukocyte Tracking	Structured Grid	Medical Imaging
Breadth-First Search*	Graph Traversal	Graph Algorithms
Stream Cluster*	Dense Linear Algebra	Data Mining
Similarity Scores*	MapReduce	Web Mining

Rodinia

Benchmark	Description
2mm	2 Matrix Multiplications (D=A.B; E=C.D)
3mm	3 Matrix Multiplications (E=A.B; F=C.D; G=E.F)
adi	Alternating Direction Implicit solver
atax	Matrix Transpose and Vector Multiplication
bicg	BiCG Sub Kernel of BiCGStab Linear Solver
cholesky	Cholesky Decomposition
correlation	Correlation Computation
covariance	Covariance Computation
doitgen	Multiresolution analysis kernel (MADNESS)
durbin	Toeplitz system solver
dynprog	Dynamic programming (2D)
fdtd-2d	2-D Finite Different Time Domain Kernel
fdtd-apml	FDTD using Anisotropic Perfectly Matched Layer
gauss-filter	Gaussian Filter
gemm	Matrix-multiply C=alpha.A.B+beta.C
gemver	Vector Multiplication and Matrix Addition
gesummv	Scalar, Vector and Matrix Multiplication
gramschmidt	Gram-Schmidt decomposition
jacobi-1D	1-D Jacobi stencil computation
jacobi-2D	2-D Jacobi stencil computation
lu	LU decomposition
ludcmp	LU decomposition
mvt	Matrix Vector Product and Transpose
reg-detect	2-D Image processing
seidel	2-D Seidel stencil computation
symm	Symmetric matrix-multiply
syr2k	Symmetric rank-2k operations
syrk	Symmetric rank-k operations
trisolv	Triangular solver
trmm	Triangular matrix-multiply

PolyBench

Questions?